GPS Error sources (USER DEPENDENT AND USER INDEPENDENT)
Ranging errors are grouped into the six following classes

1) **Ephemeris data** -- Errors in the transmitted location of the satellite including SA
2) **Satellite clock** -- Errors in the transmitted clock, including SA
3) **Ionosphere** -- Errors in the corrections of pseudorange caused by ionospheric effects
4) **Troposphere** -- Errors in the corrections of pseudorange caused by tropospheric effects
5) **Multipath** -- Errors caused by reflected signals entering the receiver antenna
6) **Receiver** -- Errors in the receiver's measurement of range caused by receiver clock error, thermal noise, software accuracy.
1) **Ephemeris data** -- Errors in the transmitted location of the satellite including SA

<table>
<thead>
<tr>
<th>Orbit type</th>
<th>Quality(m)</th>
<th>Availability</th>
<th>Available at</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broadcast</td>
<td>3.00</td>
<td>Real time</td>
<td>Receiver</td>
</tr>
<tr>
<td>Code predicted</td>
<td>.20</td>
<td>Real time</td>
<td>FTP</td>
</tr>
<tr>
<td>Code Rapid</td>
<td>.10</td>
<td>After 16hrs</td>
<td>FTP</td>
</tr>
<tr>
<td>Code Final</td>
<td>.05</td>
<td>After 5–11 days</td>
<td>FTP</td>
</tr>
<tr>
<td>IGS Ultra Rapid</td>
<td>.20</td>
<td>After 3hrs</td>
<td>IGS</td>
</tr>
<tr>
<td>IGS Rapid</td>
<td>.10</td>
<td>After 19 hrs</td>
<td>IGS</td>
</tr>
<tr>
<td>IGS Final</td>
<td>.05</td>
<td>After 13 Days</td>
<td>IGS</td>
</tr>
</tbody>
</table>
### Errors in Baseline component due to orbit error

<table>
<thead>
<tr>
<th>Orbit error In m</th>
<th>Baseline length</th>
<th>Baseline Error in ppm</th>
<th>Baseline Error in mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5 m</td>
<td>1 km</td>
<td>.1 ppm</td>
<td>- mm</td>
</tr>
<tr>
<td>2.5 m</td>
<td>10 km</td>
<td>.1 ppm</td>
<td>1 mm</td>
</tr>
<tr>
<td>2.5 m</td>
<td>100 km</td>
<td>.1 ppm</td>
<td>10 mm</td>
</tr>
<tr>
<td>2.5 m</td>
<td>1000 km</td>
<td>.1 ppm</td>
<td>100 mm</td>
</tr>
<tr>
<td>0.5 m</td>
<td>1 km</td>
<td>0.002 ppm</td>
<td>- mm</td>
</tr>
<tr>
<td>0.5 m</td>
<td>10 km</td>
<td>0.002 ppm</td>
<td>- mm</td>
</tr>
<tr>
<td>0.5 m</td>
<td>100 km</td>
<td>0.002 ppm</td>
<td>.2 mm</td>
</tr>
<tr>
<td>0.5 m</td>
<td>1000 km</td>
<td>0.002 ppm</td>
<td>2 mm</td>
</tr>
</tbody>
</table>
2) Satellite clock -- Errors in the transmitted clock, including SA

Satellite clock error correction for point positioning

-- IGS clock corrections

-- broadcast clock corrections

-- broadcast group delay correction
2) **Satellite clock** -- Errors in the transmitted clock, including SA

Satellite clock error correction for relative positioning

Single difference of the Carrier phase pseudo ranges eliminate the satellite clock bias
3) **Ionosphere** -- Errors in the corrections of pseudorange caused by ionospheric effects

Ionosphere which extends from a height of 70 to 1000km

Dispersive medium and effects the GPS radio signal

- *Carrier experiences a phase advance*
- *Code experiences a group delay*

**Delay is dependent on Total Electronic Count (TEC)**

- *Larger delay from low elevation satellites*
- *Peaking in the day time and subsiding during the night (solar radiation effect)*
- *Near the geomagnetic equator/poles the delays are larger*

Ionosphere delay is frequency dependent (proportional to $1/f^2$)

$f$ is the carrier frequency
3) **Ionosphere** --Errors in the corrections of pseudorange caused by ionospheric effects

Error analysis of GPS point and relative positioning

**Dual frequency correction (L1/L2) for point positioning and short baselines**

**Ionosphere free linear combination of L1 and L2 for long baselines**

\[
L_3 = \frac{1}{f_1^2 - f_2^2} \left( f_1^2 L_1 - f_2^2 L_2 \right)
\]

\[
L_6 = \frac{1}{f_1 - f_2} \left( f_1 L_1 - f_2 L_2 \right) - \frac{1}{f_1 \pm f_2} \left( f_1 P_1 \pm f_2 P_2 \right)
\]

**IGS ionosphere model correction**

**broadcast (Klobuchar) model correction**

**no correction (single frequency receivers)**
3) Ionosphere error- Options Available

**IGNORE** the bias -- makes cycle slip editing and ambiguity resolution more difficult, and also introduces scale errors, *though only significant for long baselines.*

**OBSERVE AT NIGHT** if possible -- during minimum ionospheric activity.

Use **IONOSPHERE PREDICTION MODELS** -- broadcast model generally <50% accuracy, *may be useful for point positioning users.*

Use **DUAL-FREQUENCY** receivers -- form "ionosphere-free" L1/L2 data combination:

**DIFFERENCE** data between sites -- effect of error is minimised due to its high correlation over short to medium baselines, *typically 1-2ppm residual effect.*
4) **Troposphere**--Errors in the corrections of pseudorange caused by tropospheric effects

Effect of Neutral atmosphere (nonionized part) 0 to 70 km
Non dispersive medium for the GPS radio signals
Causes delay in both code and carrier observations
Frequency independent
Effect is same for phase and code measurements
Dry component causes 90% of the total delay
Wet part causes 10% of the total delay
Systematic error
4) **Troposphere**--Errors in the corrections of pseudorange caused by tropospheric effects

- Since it a frequency independent error, cannot be cancelled using dual frequency measurements
- Linear combination which eliminates the troposphere error

\[
L_6 = \frac{1}{f_1 - f_2} (f_1 L_1 - f_2 L_2) - \frac{1}{f_1 \pm f_2} (f_1 P_1 \pm f_2 P_2)
\]

- Troposphere can be successfully modeled using the values of temperature, pressure and relative humidity and satellite elevation angle.
4) **Troposphere**--Errors in the corrections of pseudorange caused by tropospheric effects

- Models by Hopfield, Black and Saastamoninen predict the dry part delay to 1 cm and wet part to 5 cm

- Alternatively the troposphere parameters can be estimated for each site in post processing mode and the correction applied.

- Modelling of troposphere effect is still a dynamic research area and will be so for some more years.
4) **Troposphere** -- Errors (OPTIONS AVAILABLE)

**IGNORE** the bias -- but avoid tracking low elevation satellites, generally no observations taken below 20°.

**CORRECT** data using a **STANDARD TROPOSPHERIC REFRACTION MODEL** (Saastamoinen, Hopfield, etc.) -- with or without surface meteorological readings.

**ESTIMATE** residual tropospheric effect as an additional parameter -- only for very high precision work

**DIFFERENCE** data between sites -- effect of error is minimised due to high correlation over short to medium baselines.

**MEASURE** wet path component directly using a Water Vapour Radiometer -- too expensive, no longer considered a viable option.
5) **Multipath**--Errors caused by reflected signals entering the receiver antenna

- Reflection of GPS signal by some object/surface before it is tracked by the receiver
- Reflections due to surfaces surrounding the antenna ~ 15cm for the L1 carrier and 15-20m for pseudoranges
- Care should be taken that there no reflecting surfaces around the antenna
- The observations are usually marked during processing
- Multi-path causes cycle slips which during processing are detected and repaired.
6) **Receiver**--Errors in the receiver's measurement of range caused by receiver clock, thermal noise, software accuracy.

In point positioning more precise carrier observation reduce the error to 50% Double difference of the Carrier phase pseudo ranges eliminate the satellite clock bias

**Relative positioning – Double differences**

![Diagram of relative positioning](image)
ANTI SPOOFING (AS)

AS prevents the receiver to make P-code measurements. Data corruption under AS has little or no effect as the post processing and receivers take care of it. 2-3 mm horizontal repeatability

AS will most directly affect kinematic, rapid static
SELECTIVE AVAILABILITY

Dither

Epsilon

Reported position
Actual position
C/A code ≠ 293m
P code 9 29.3m

Satellite with SA
Satellite without SA

Time

PRC
Comparison of Positions With and Without SA Full 24 Hour Data Sets

Point scatter of 24 hrs data at continuously operating site (http site http://www.ngs.noaa.gov/FGCS/info/sans_SA/)
Miscellaneous GPS errors

Geometric errors (GDOP)

- GPS ranging errors are magnified due to range vector differences between the receiver and the SVs

Cycle Slips

- Obstruction of Satellite Signals
- Bad Ionosphere Condition
- Multi Path
- Satellite Elevation
- Receiver software failure
- Satellite oscillator malfunctioning

Ambiguity resolution

- Effects the accuracy of the position of the receiver
POOR GDOP
GOOD GDOP
GOOD GDOP - BAD VISIBILITY
GDOP terms are usually computed using parameters from the
navigation solution process

\[
\text{Satellite (SV) coordinates in ECEF XZY from Ephemeris Parameters and SV Time}
\]

\[
\begin{align*}
\text{SVx}_0 &= 15524471.175 \quad \text{SVy}_0 &= -16649626.222 \quad \text{SVz}_0 &= 13512272.387 \quad \text{SV 15} \\
\text{SVx}_1 &= -2304058.534 \quad \text{SVy}_1 &= -23287906.465 \quad \text{SVz}_1 &= 11917038.105 \quad \text{SV 27} \\
\text{SVx}_2 &= 16680243.357 \quad \text{SVy}_2 &= -3069625.561 \quad \text{SVz}_2 &= 20378551.047 \quad \text{SV 31} \\
\text{SVx}_3 &= -14799931.395 \quad \text{SVy}_3 &= -21425358.24 \quad \text{SVz}_3 &= 6069947.224 \quad \text{SV 7}
\end{align*}
\]

\[
\text{Satellite Pseudorange in meters (from C/A code epochs in milliseconds)}
\]

\[
\begin{align*}
P_0 &= 89491.971 \\
P_1 &= 133930.500 \\
P_2 &= 283098.754 \\
P_3 &= 205961.742
\end{align*}
\]

\[
\text{Range + Receiver Clock Bias}
\]

\[
\text{Receiver Position Estimate in ECEF XZY}
\]

\[
\begin{align*}
\text{Rx} &= -730000 \\
\text{Ry} &= -54440000 \\
\text{Rz} &= 3230000
\end{align*}
\]

\[
\text{For Each of 4 SVs}
\]

\[
\begin{align*}
\text{Ranges from Receiver Position Estimate to SVs (R)} &\quad \text{Array of Observed - Predicted Ranges}
\end{align*}
\]

\[
\begin{align*}
R_i &= \sqrt{\left(\text{SVx}_i - \text{Rx}\right)^2 + \left(\text{SVy}_i - \text{Ry}\right)^2 + \left(\text{SVz}_i - \text{Rz}\right)^2} \\
L_i &= \text{mod}\left(\left(R_i\right), 299792.458\right) - P_i
\end{align*}
\]

\[
\text{Compute Directional Derivatives for XZY and Time}
\]

\[
\begin{align*}
\text{Dx}_i &= \frac{\text{SVx}_i - \text{Rx}}{R_i} \\
\text{Dy}_i &= \frac{\text{SVy}_i - \text{Ry}}{R_i} \\
\text{Dz}_i &= \frac{\text{SVz}_i - \text{Rz}}{R_i} \\
\text{Dt}_i &= -1
\end{align*}
\]

\[
\text{Solve for Correction to Receiver Position Estimate}
\]

\[
A := \begin{bmatrix}
\text{Dx}_0 & \text{Dy}_0 & \text{Dz}_0 & \text{Dt}_0 \\
\text{Dx}_1 & \text{Dy}_1 & \text{Dz}_1 & \text{Dt}_1 \\
\text{Dx}_2 & \text{Dy}_2 & \text{Dz}_2 & \text{Dt}_2 \\
\text{Dx}_3 & \text{Dy}_3 & \text{Dz}_3 & \text{Dt}_3
\end{bmatrix}
\]

\[
dR := (A^T \cdot A)^{-1} \cdot A^T \cdot L
\]

\[
dR = \begin{bmatrix}
-3186.496 \\
-3791.932 \\
1193.286 \\
12345.997
\end{bmatrix}
\]

\[
\text{Apply Corrections to Receiver XYZ and Compute Receiver Clock Bias Estimate}
\]

\[
\begin{align*}
\text{Rx} &= \text{Rx} + dR_x \\
\text{Ry} &= \text{Ry} + dR_y \\
\text{Rz} &= \text{Rz} + dR_z \\
\text{Time} &= \text{Time} + dR_t
\end{align*}
\]

\[
\begin{align*}
\text{Rx} &= -733186.496 \\
\text{Ry} &= -5443791.932 \\
\text{Rz} &= 3231193.286 \\
\text{Time} &= 12345.997
\end{align*}
\]
GDOP Computation example

GPS GDOP Example - Peter H. Dana - 4/24/96

Satellite (SV) coordinates in ECEF XYZ from Ephemeris Parameters and SV Time

<table>
<thead>
<tr>
<th>SV</th>
<th>SVx</th>
<th>SVy</th>
<th>SVz</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15524471.175</td>
<td>-16649826.222</td>
<td>13512272.387</td>
</tr>
<tr>
<td>1</td>
<td>-2304058.534</td>
<td>-23287906.465</td>
<td>11917038.105</td>
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<tr>
<td>3</td>
<td>-14799931.395</td>
<td>-21425358.24</td>
<td>606947.224</td>
</tr>
</tbody>
</table>

SV 0, 15
SV 1, 27
SV 2, 31
SV 3, 7

Receiver Position Estimate in ECEF XYZ

Rx := -730000
Ry := -5440000
Rz := 3230000

For Each of 4 SVs

For Each of 4 SVs

Ranges from Receiver Position Estimate to SVs (R) and Array of Observed - Predicted Ranges

\[ R_i := \sqrt{(SVx_i - Rx)^2 + (SVy_i - Ry)^2 + (SVz_i - Rz)^2} \]

Compute Directional Derivatives for XYZ and Time

\[ D_x_i := \frac{SVx_i - Rx}{R_i}, \quad D_y_i := \frac{SVy_i - Ry}{R_i}, \quad D_z_i := \frac{SVz_i - Rz}{R_i}, \quad D_t_i := -1 \]

Solve for Correction to Receiver Position Estimate

\[ A := \begin{bmatrix} D_x_0 & D_y_0 & D_z_0 & D_t_0 \\ D_x_1 & D_y_1 & D_z_1 & D_t_1 \\ D_x_2 & D_y_2 & D_z_2 & D_t_2 \\ D_x_3 & D_y_3 & D_z_3 & D_t_3 \end{bmatrix}, \quad P := (A^T A)^{-1} \]

Compute Geometric Dilution of Precision (GDOP) terms:

\[ \text{GDOP} := \sqrt{P_{0,0} + P_{1,1} + P_{2,2} + P_{3,3}} \]

GDOP = 6.806

\[ \text{PDOP} := \sqrt{P_{0,0} + P_{1,1} + P_{2,2}} \]

PDOP = 6.171

\[ \text{TDOP} := \sqrt{P_{3,3}} \]

TDOP = 2.871
GDOP COMPONENTS

PDOP = Position Dilution of Precision (3-D), sometimes the Spherical DOP.

HDOP = Horizontal Dilution of Precision (Latitude, Longitude).

VDOP = Vertical Dilution of Precision (Height).

TDOP = Time Dilution of Precision (Time).

While each of these GDOP terms can be individually computed but they are not independent of each other. Linear combination of both phase and code observables eliminates the geometric errors

\[ L_6 = \frac{1}{f_1 - f_2} \left( f_1 L_1 - f_2 L_2 \right) - \frac{1}{f_1 \pm f_2} \left( f_1 P_1 \pm f_2 P_2 \right) \]
Effect on measurement due to CYCLE SLIPS
A "jump" the sequence of carrier phase measurements due to the occurrence of a cycle slip
• Cycle slips are usually **detected** and **repaired** in a data pre-processing step.

• Linear combination at the double difference level is used for cycle slip fixing.

\[ L_5 = \frac{1}{f_1 - f_2} (f_1 L_1 - f_2 L_2) \]

• Cycle slip *detection* is easier than its *correction*, especially if the slip is "large" (say, greater than about 10 cycles).

• Cycle slip *correction* involves the determination of the **exact** number of slipped carrier cycles at epoch \( T_e \).
• Cycle slip correction is easier when the clock biases have been eliminated during between-receiver and/or between-satellite differencing.

• Cycle slip correction of dual-frequency data is easier than for single frequency data.

• Cycle slip correction of static data is easier than the case of kinematic data.

• Cycle slip correction of data in the post-mission mode is easier than for the case of real-time data processing.
The phase ambiguity term and a range measurement
1. "PHASE MEASUREMENT" : (SAY NOW=1/4)

= FRACTION OF WHOLE WAVELENGTH X SIGNAL
WAVELENGTH = 1/4 x 0.190M = 0.0475M

2. "INITIAL AMBIGUITY AT FIRST OBSERVATION" :

= NUMBER OF FULL WAVELENGTHS X SIGNAL
WAVELENTH = 106,000,000 x 0.190M = 20,140,000M

3. "NUMBER OF FULL CYCLES COUNTED BY RECEIVER" :

= NUMBER OF COUNTED WAVELENGTHS X SIGNAL
WAVELENGTH = 1,000 x 0.190M = 190M

TOTAL DISTANCE TO SV:

1 + 2 + 3 = 0.0475 +20,140,000 +190 = 20,140,190.0475M
Effect of Phase ambiguity Bias

**EFFECT:**

• The existence of phase ambiguities means that instantaneous positioning using phase data, at a single epoch, is not possible.

• Including ambiguities as extra estimable parameters makes for a more complex positioning solution.

• Theoretically all phase ambiguities should be integer values.
Resolving ambiguities

• Using Single frequency phase data - models are used

• Using dual frequency phase data - Linear combination of L1 and L2 at the double difference level is used

\[ L_5 = \frac{1}{f_1 - f_2} (f_1 L_1 - f_2 L_2) \]

• Using dual frequency carrier phase and code data - Linear combination of both carrier phase and code observables is used

\[ L_6 = \frac{1}{f_1 - f_2} (f_1 L_1 - f_2 L_2) - \frac{1}{f_1 \pm f_2} (f_1 P_1 \pm f_2 P_2) \]
OPTIONS AVAILABLE

• **ADJUST** all ambiguities as free parameters, together with station coordinates, in a so-called *ambiguity-free* solution.

• "**RESOLVE**" estimated ambiguity parameters to their nearest integer values, and iterate solution, in a so-called *ambiguity-fixed* solution -- in effect use precise unambiguous phase-range data.

• **DIFFERENCE** data between consecutive epochs to **ELIMINATE** the phase ambiguity bias.

• The strongest baseline solution is provided by "carrier-range" (or "phase-range") double-differenced data, formed when ambiguous carrier phase data is "corrected" so as to generate a high precision range-like observable.